 LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**M.Sc.,** DEGREE EXAMINATION - **MATHEMATICS**

# SECOND SEMESTER – APRIL 2012

# MT 2810/MT 2804 – ALGEBRA

Date :17-04-2012 Dept. No. Max. : 100 Marks

Time: 9.00-12.00

Answer **ALL** the Questions

1. a) If G is a finite group, then prove that  in other words, show that the number of

elements conjugate to a in G is the index of the normalizer of a in G.

**(OR)**

b) If p is a prime number and , then show that G has an element of order p. (5)

c) Prove that G has a subgroup of order where p is a prime number and  divides O (G).

**(OR)**

d) Prove that every finite abelian group is the direct product of cyclic groups. (15)

1. a) Given two polynomials and in  then prove that there exist two polynomials

and  in such that where or .

**(OR)**

1. If the primitive polynomial can be factored as the product of two polynomials having rational coefficients prove that it can be factored as the product of two polynomials having integer coefficients. (5)
2. (i) State and prove the Eisenstein criterion.

(ii) State and prove Gauss lemma.

**(OR)**

1. Let R be a Euclidean ring, then show that any finitely-generated R-module M is the direct sum of a finite number of cyclic submodules. (15)
2. a) If L is an algebraic extension of K and K is an algebraic extension of F, prove that L is an

algebraic extension of F.

**(OR)**

1. If  is a polynomial in F[x] of degree and is irreducible over F, then prove that there is an extension E of F, such that [E : F] = n in which  has a root. (5)
2. The element a ∈ K is said to be algebraic over F iff F(a) is a finite extension of F.

**(OR)**

1. (i) If a, b in K are algebraic over F then show that a ± b, ab and are algebraic over F.

(ii) If F is of characteristic 0 and a, b are algebraic over F, then show that there exists an

element c ∈ F(a,b) such that F (a,b) = F(c). (15)

1. a) Find the fixed field of G (K, F) where F is the field of real numbers and K is the field of

complex numbers.

**(OR)**

b) Prove that S4 is solvable. (5)

c) State and prove the fundamental theorem of Galois theory.

**(OR)**

d) K is the normal extension of F iff K is the splitting field of some polynomial over F. (15)

1. a) Let G be a finite abelian group such that the relation satisfied by atmost n elements of G for every positive integer n then prove that G is a cyclic group.

**(OR)**

b) For every prime number p and for every positive integer m, prove that there is a unique field

having pm elements. (5)

c) State and prove Wedderburn’s theorem on finite division rings.

**(OR)**

d) Let K be the normal extension of F and H  G (K, F), is the

fixed field of H then prove that

(i) [K : KH]= O (H)

(ii) H = G (K, KH). In particular, H = G (K,F) and [K : F] = O . (15)

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